3/15/18

"The question isn't 'Who is going to let me?'; It's 'Who is going to stop me?'." -Ayn Rand

HW: "Indefinite Integral" #1-6, 15-18
Test 3 on Wednesday 3/28

AIM: What is Integration?

ANTIDERIVATIVES AND INDEFINITE INTEGRATION

VOCABULARY AND DEFINITIONS:

1. A function $F(x)$ is an *antiderivative* of a function if $F'(x) = f(x)$ for all x in domain of f. The process of finding an antiderivative is *antidifferentiation*.

2. The family of all antiderivatives of a function $f(x)$ is the indefinite integral of f with respect to x and is denoted by $\int f(x)\,dx$

3. If F is any function such that $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + C$ is called the *general solution* and C is called the *constant of integration* (an arbitrary constant).

General Solution is denoted by:

Variable of Integration

Constant of Integration

$$y = \int f(x)\,dx = F(x) + C$$

Integrand

$\int f(x)\,dx$ read as the *antiderivative of f with respect to x.*

So, the differential *dx* serves to identify *x* as the variable of integration. The term indefinite integral is a synonym for antiderivative.

EX #3: Applying Basic Rules

A.) $\int 3x^{1}dx = \boxed{\dfrac{3x^2}{2} + c}$

B.) $\int 8\,dx = \boxed{8x + c}$

C.) $\int (x^2 - 3x + 4)dx = \boxed{\dfrac{x^3}{3} - \dfrac{3x^2}{2} + 4x + c}$

EX #4: Rewriting Before Integrating

Original Integral	Rewrite	Integrate	Simplify
$\int \frac{1}{x^3}\,dx$	$\int x^{-3}\,dx$	$\frac{x^{-2}}{-2} + c$	$\frac{1}{-2x^2} + c$
$\int \sqrt{x}\,dx$	$\int x^{1/2}\,dx$	$\frac{x^{3/2}}{3/2} + c$	$\frac{2}{3}\sqrt{x^3} + c$
$\int 2\sin x\,dx$	$2\int \sin x\,dx$	$2(-\cos x) + c$	$-2\cos x + c$

Recall:
Derivative

$$3x^2 \downarrow 6x$$

$$3 \cdot x^2$$

$$3 \cdot 2x$$

$$6x$$

When integrating, think before you work! Practice will help you “discover” many tricks that make the integrations rules “fit your problems.”

TRICK #1 REWRITE	TRICK #2 MULTIPLE or DISTRIBUTE	TRICK #3 SEPARATE FRACTIONS

EX #5: Integrate the following:

A.) $\int (3x-1)^2\,dx$

$\int (3x-1)(3x-1)\,dx$

$\int 9x^2-6x+1\,dx$

$= \frac{9x^3}{3} - \frac{6x^2}{2} + \frac{1x}{1} + c$

$= \boxed{3x^3 - 3x^2 + x + c}$

B.) $\int \frac{x^2+4x+5}{x^5}\,dx$

$\int \frac{x^2}{x^5} + \frac{4x}{x^5} + \frac{5}{x^5}\,dx$

$\int x^{-3} + 4x^{-4} + 5x^{-5} = \frac{x^{-2}}{-2} + \frac{4x^{-3}}{-3} + \frac{5x^{-4}}{-4} + c$

$= \boxed{\frac{1}{-2x^2} - \frac{4}{3x^3} - \frac{5}{4x^4} + c}$